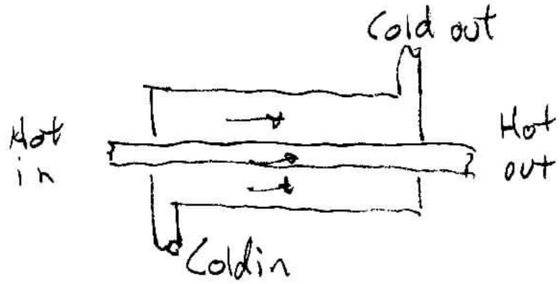
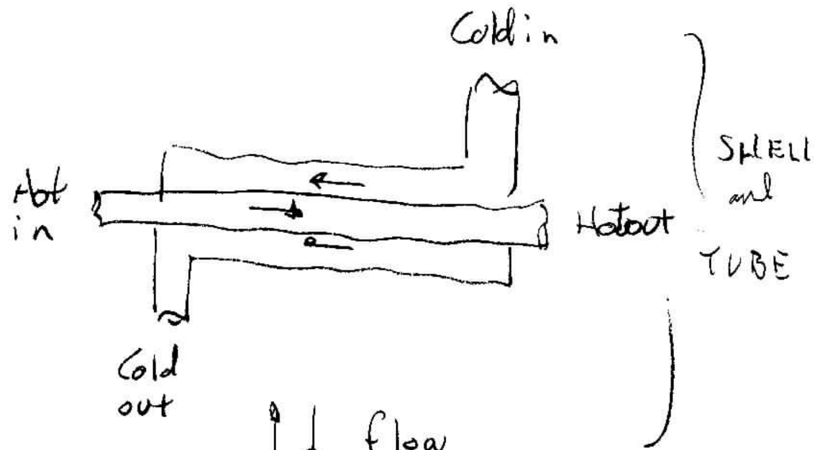
Heat Exchangers



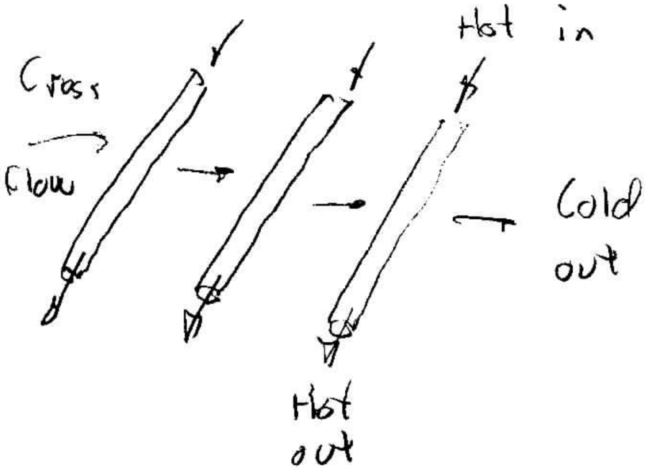
⇈ flow

parallel flow

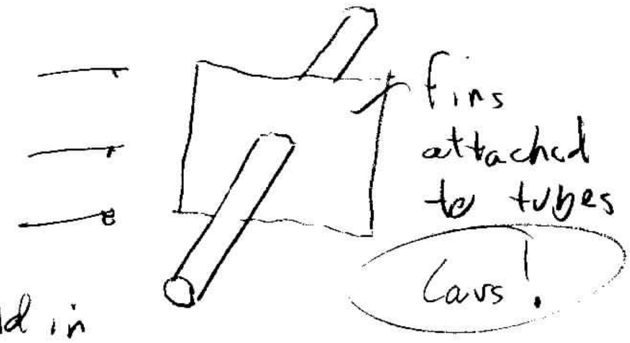


↑↓ flow

Counter flow

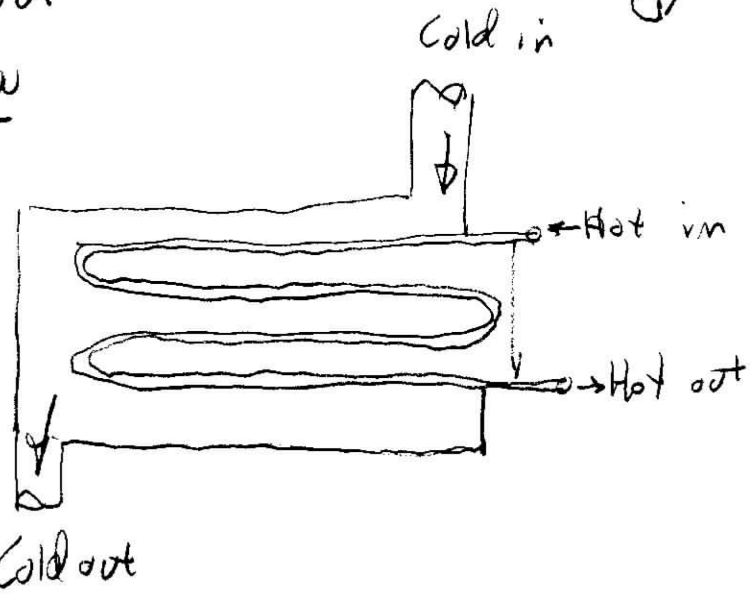


or



Cross flow

Multi pass

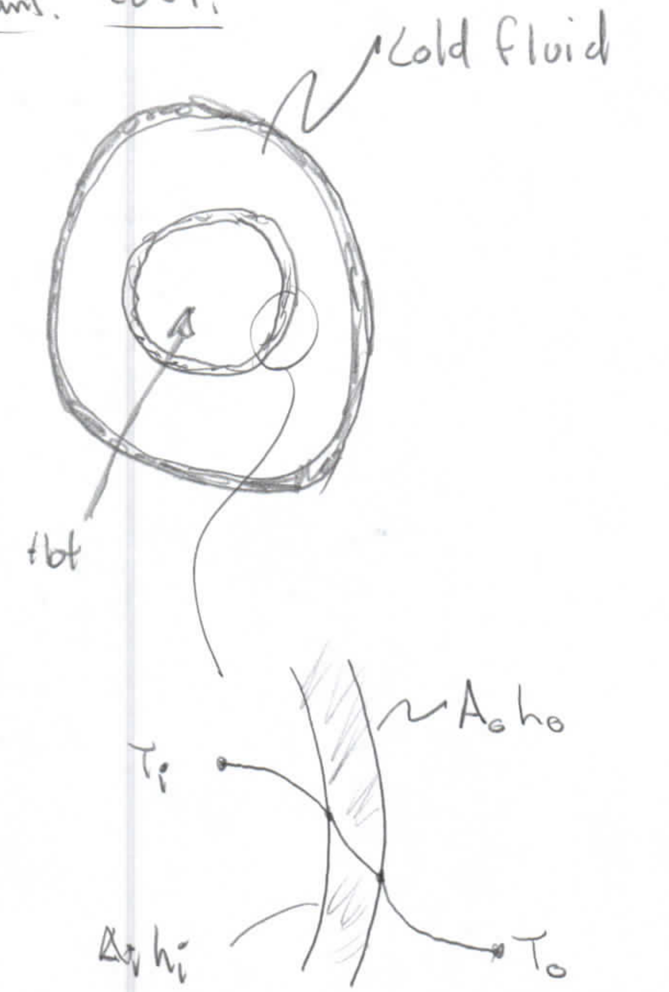


Let your imagination run free!

Don't forget fins/pins.

Over all Heat transf. coef.

Consider concentric tubes



T_i ———— T_o

$R_i = \dfrac{1}{h_i A_i}$ $\quad R_{wall} \quad$ $R_o = \dfrac{1}{h_o A_o}$

Probably:

$A_i = \pi D_i L$

$A_o = \pi D_o L$

$$R_{wall} = \frac{\ln(D_o/D_i)}{2\pi k L}$$

k ~ cond. of tube

L ~ length of tube

The total resistance is

$$R_{total} = R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}$$

Recall overall heat transfer coefficient U

$$q = \frac{\Delta T}{R_{total}} = U A_s \Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

in which case

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R_{total} = \frac{1}{h_iA_i} + R_{wall} + \frac{1}{h_oA_o}$$

so $U_iA_i = U_oA_o$

but does not always mean $U_i = U_o$ (unless $A_i = A_o$)

If R_{wall} is negl. (thin, high cond. material)

then $\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$

you can find tables for

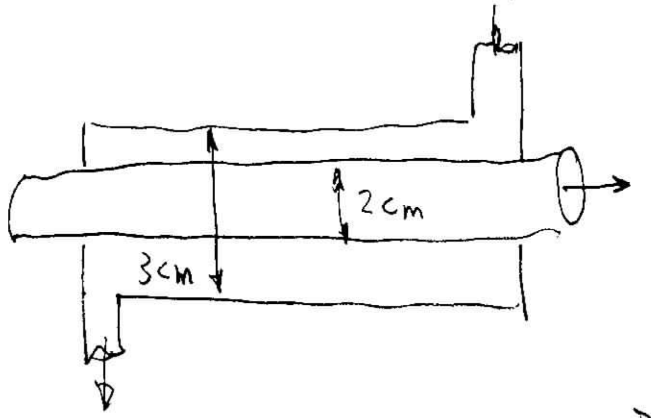
$U\left[\frac{W}{m^2K}\right]$	Type of hx

R_f ~ Fouling Factors (on outer surface of walls)

$$\frac{1}{UA_s} = R_{total} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

Look up in tables.
Usually account for
them when equipment
is new.

Ex 1 Cool hot oil in tube/shell h.x.



Determine

T_{avg} water is 45°C

T_{avg} oil is 80°C

Assume:
- negl R_{tube} (↑ k, thin wall)
- Both water + oil are fully developed
- matl. prop. are ¢

Prop. of water @ 45°C

$\rho = 990.1\ \frac{kg}{m^3}$ $Pr = 3.91$

$k = 0.637\ \frac{W}{m \cdot K}$ $\nu = \frac{\mu}{\rho} = 0.602 \times 10^{-6}\ m^2/s$

Prop. of oil @ 80°C

$\rho = 852\ \frac{kg}{m^3}$ $Pr = 499.3$

$k = 0.138\ \frac{W}{m\,k}$ $\nu = 3.794 \times 10^{-5}\ m^2/s$

Lets go...

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$ so we need to calc these...

Hydraulic diam of tube $D_{h\,tube} = D_{tube} = 0.02\ m$

average vel. of water $\quad V = \dfrac{\dot{m}_w}{\rho A_c} = \dfrac{\dot{m}_w}{\rho_w\left(\frac{\pi}{4}D^2\right)} = \ldots = 1{,}61 \ \mathrm{m/s}$

in which case $\quad Re\Big|_{water} = \dfrac{VD}{\nu}\Big|_{water} = \ldots = 53{,}490 > 10{,}000$ (crit for turb.)

$\Rightarrow$ water is turb.

Full dev, turb: $\quad Nu = \dfrac{hD}{k}\Big|_{water} = 0{,}023\, Re^{0{,}8}\, Pr^{0{,}4} = \ldots = 240{,}6$

so that $\quad h_{water} = \dfrac{k}{D}\Big|_{water} Nu = 7663 \ \mathrm{W/m^2K}$

Do this for the oil ... $\quad D_{hyd} = \underbrace{D_o - D_i}_{\text{annular section!}} = (0{,}03 - 0{,}01)\ \mathrm{m} = 0{,}01\ \mathrm{m}$

Avg velocity of oil $\quad V = \dfrac{\dot{m}_{oil}}{\rho A_c} = \dfrac{\dot{m}_{oil}}{\rho\left(D_o^2 - D_i^2\right)\frac{\pi}{4}} = \ldots = 2{,}39 \ \mathrm{m/s}$

so then $\quad Re\Big|_{oil} = \dfrac{VD}{\nu}\Big|_{oil} \simeq 630 < 2300$ (crit for lam)

$\Rightarrow$ oil is laminar

Fully dev lam: in annular section $\quad \dfrac{D_i}{D_o} = \dfrac{0{,}02}{0{,}03} = 0{,}667$ (see table)

$Nu = 5{,}45$

thus $h_{oil} = \frac{k}{D_h} Nu_{oil} = 75.2 \frac{W}{m^2 K}$

Finally

$$\frac{1}{U_{net}} = \underbrace{\frac{1}{h_{i,water}}}_{\substack{\frac{1}{7663 \frac{W}{m^2 K}} \\ \text{negl.}}} + \underbrace{\frac{1}{h_{oil}}}_{\substack{\frac{1}{75.2 \, W/m^2 K} \\ \text{important}}} \Rightarrow U_{net} = 74.5 \frac{W}{m^2 K}$$

thus $U_{net} = h_o$

- overall heat transfer is dominated by the (much) smaller h_o. Bottle neck for heat transfer!

<u>Analysis</u> of heat transfer

LMTD	effectiveness
log mean	NTU
T-diff	method
(today)	(recorded)

↳ Assume S.S.

- Inlet/outlet cond. are ¢ in time
- matl. prop are ¢ in time
- <u>Neglect</u> <u>axial</u> <u>heat</u> cond.
- <u>Perfect</u> <u>insulation</u> on out side of h.x.

At any section of hot and cold fluid

$$q_{\text{hot to cold}} = q_{\text{cold to hot}}$$

$$\dot{m} c_p (T_{out} - T_{in})\Big|_{cold} = \dot{m} c_p (T_{in} - T_{out})\Big|_{hot}$$

Just keep <u>q</u> <u>Inherently</u> <u>$\oplus$</u> and refer to direction (to or from)

You see $\dot{m} c_p$ all over the place.

They often let $C_h = \dot{m} c_p\Big|_{hot}$ $\quad C_c = \dot{m} c_p\Big|_{cold}$

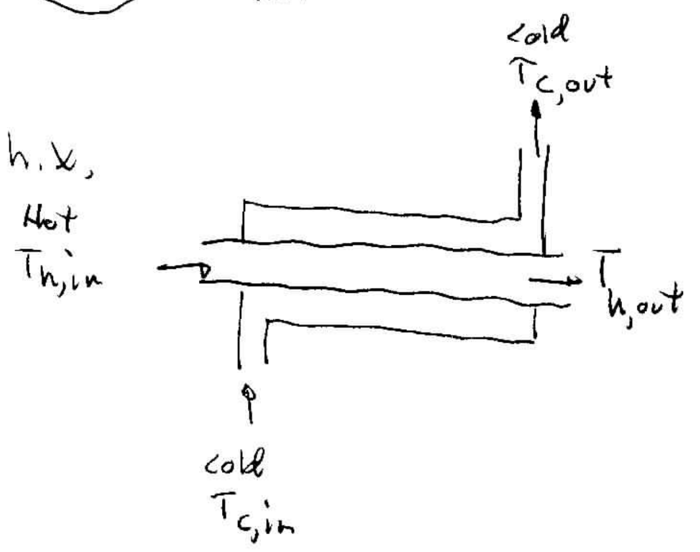
Thus

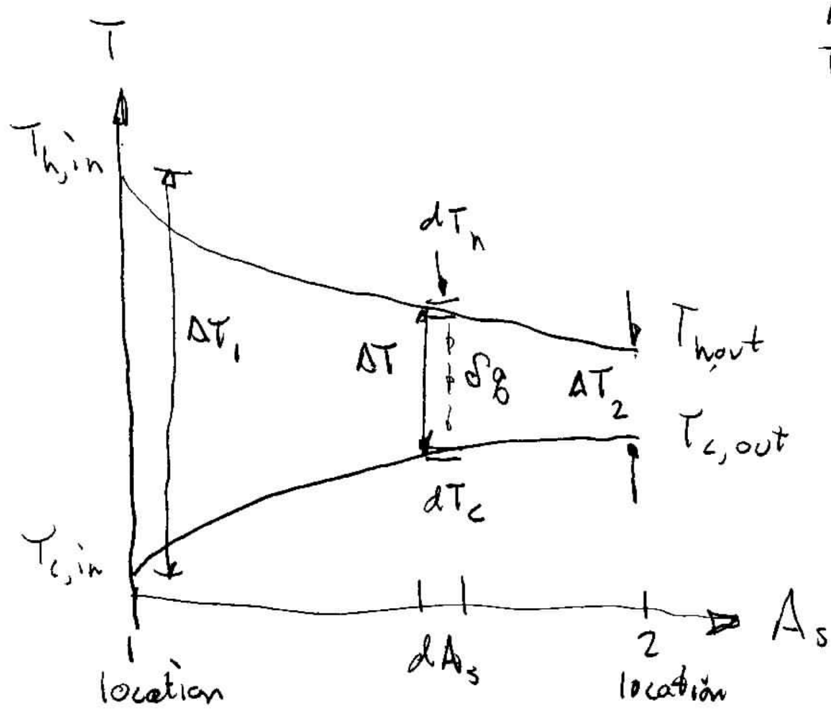
$$q = C (T_{out} - T_{in})\Big|_{cold} = C (T_{in} - T_{out})\Big|_{hot}$$

SUE $\qquad\qquad$ BOB

<u>LMTD</u> <u>method</u>

consider ↑↑ flow double tube h.x.





Neglecting changes in K.E. P.E. we only exchange T.E.

so $$\delta q = -\dot m_h c_{p,h}\, dT_h = +\dot m_c c_{p,c}\, dT_c$$

in which case $dT_h = \dfrac{-\delta q}{\dot m_h c_{ph}}$ $\quad \delta T_c = \dfrac{+\delta q}{\dot m_c c_{pc}}$

Subtract to get $$dT_h - dT_c = d(T_h - T_c) = -\delta q \left(\frac{1}{\dot m c_p}\Big|_h + \frac{1}{\dot m c_p}\Big|_c \right)$$

But the rate of heat transfer in a section is also

$$\delta q = U(dA_s)(T_h - T_c)_{local}$$

put this in here and rearrange

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U\left(\frac{1}{\dot m c_p}\Big|_h + \frac{1}{\dot m c_p}\Big|_c \right) \text{ at any section}$$

$\int$ from inlet to outlet to get

$$\ln\left[\frac{(T_h - T_c)_{out}}{(T_h - T_c)_{in}}\right] = -UA_s \underbrace{\left(\frac{1}{\dot m c_p}\Big|_{hot} + \frac{1}{\dot m c_p}\Big|_{cold} \right)}$$

replace with SOE + BOB to get

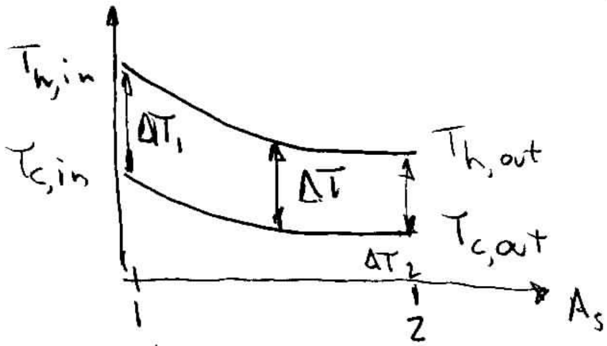
$$q = UA_s\, \Delta T_{lm} = UA_s\left[\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}\right]$$

Comments - does not matter what you consider to inlet or outlet.

- we will have a correction factor for multi-pass h.x.

- Don't use $\Delta T_{\text{arit. mean}}$! Just don't go there.

- Get identical results for ↑ h.x. Try it!



Interesting point for ↓↑ h.x. if $C_h = C_c$

$\dot{m} c_p \big|_{hot} = \dot{m} c_p \big|_{cold}$

then $\Delta T = ¢$

in which case $\Delta T_1 = \Delta T_2$

and $\Delta T_{lm} = \text{"}\frac{0}{0}\text{"}$ oops.

L'Hopital to the rescue ... will get $\Delta T_{lm} = \Delta T_1 = \Delta T_2$

whew!

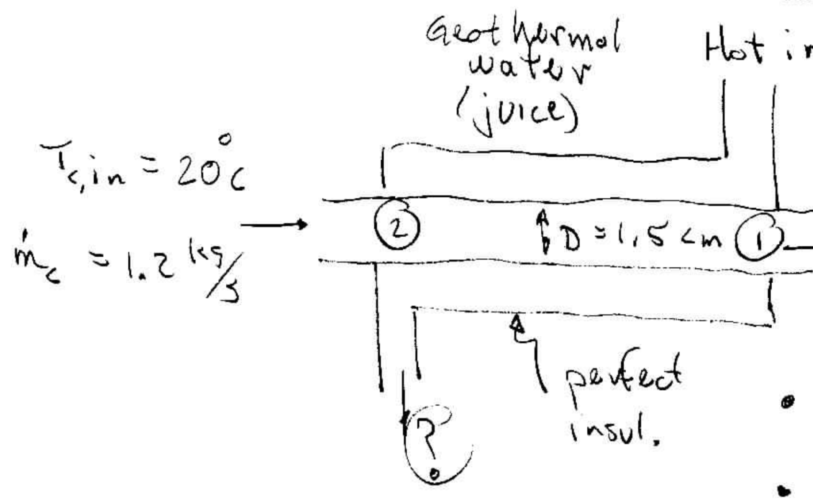
Multi-pass ↑↑ or ↑↓ h.x. and × flow:

Use $\Delta T_{lm} = F \, \Delta T_{lm, C.F.}$

↑ lots of tables + charts .. multi-pass cross-flow

- read the directions

Ex1



- $h_{overall} = 640\ W/m^2K$
- Thin wall low R inner pipe

- How long a pipe to heat our water from 20°C to 80°C?

Assume: S.S.

well insulated outer jacket

K.E. P.E. changes are negl.

No fouling

ϕ fluid properties

Prop $C_p|_{water} = 4.18\ \frac{kJ}{kg K}$

$C_p|_{juice} = 4.31\ \frac{kJ}{kg K}$

Analysis over all rate of heat exchange is

$$q = \dot{m} C_p (\underset{80°C}{T_{out}} - \underset{20°C}{T_{in}})\Big|_{water} = 301\ kW$$

Use this to calc $T_{out,juice}$

$$q = \dot{m} C_p (\underset{\text{find this}}{T_{out}} - \underset{160°C}{T_{in}})\Big|_{juice} \Rightarrow T_{out\ juice} = T_{in} - \frac{q}{\dot{m} C_p}\Big|_{juice}$$

$$= 125°C$$

so now $\Delta T_{①} = T_{h,i} - T_{c,o} = (160 - 80)°C = 80°C$

$\Delta T_{②} = T_{h,out} - T_{c,in} = (125 - 20)°C = 105°C$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} = \ldots = 91.9\,°C$$

Now calculate A_{surf} ...

$$q = U A_s \Delta T_{lm} \quad \text{so} \quad A_s = \frac{q}{U \Delta T_{lm}} = 5.12\ m^2$$

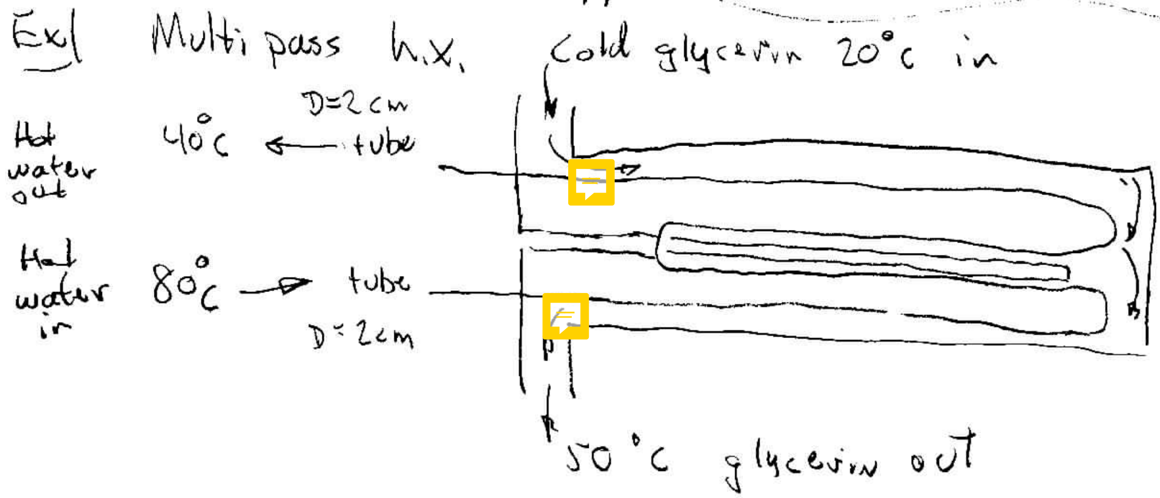
U: $646 \frac{W}{m^2 K}$

But $A_s = \pi D L \Rightarrow L = \frac{A_s}{\pi D} = 109\ m$

Comments: 109 m is too long.

Use a plate h.x. or multi-pass of tube bundles.

Ex) Multi pass h.x.



- 4-tube
  2-shell
- Total tube length = 60 m
- $h = 25 \frac{W}{m^2 K}$ on glycerin side
- $h = 160 \frac{W}{m^2 K}$ on water side

Determine q in h.x.
for a) no fouling
b) with fouling $0.0006 \frac{m^2 K}{W}$ on outer surface of tubes

Analysis · Thin walled tubes ⇒ surface area inside ≈ surface area outside

then $A_s = \pi D L = 3.77\ m^2$

$D = 0.02\,m$, $L = 60\,m$

We will use $q = U A_s F\, \Delta T_{lm,\,C.F.}$

We know $\Delta T_1 = T_{h,in} - T_{c,out} = (80-50)^\circ C = 30^\circ C$

$\Delta T_2 = T_{h,out} - T_{c,in} = (40-20)^\circ C = 20^\circ C$

so

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = 24.7^\circ C$$

counter flow

See multipass factor F calc.

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{(40-80)^\circ C}{(20-80)^\circ C} = 0.67$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(20-50)^\circ C}{(40-80)^\circ C} = 0.75$$

$F = 0.91$

a) No fouling factor

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{160\,\frac{W}{m^2K}} + \frac{1}{25\,\frac{W}{m^2K}} \Rightarrow U = 21.6\ \frac{W}{m^2K}$$

so $q = U A_s F\, \Delta T_{lm,CF} = \ldots = 1830\ W$

b) With fouling factor

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R_{foul} = \frac{1}{160\,\frac{W}{m^2K}} + \frac{1}{25\,\frac{W}{m^2K}} + 0.0006\,\frac{m^2K}{W} = 21.3\ \frac{W}{m^2K}$$

and $q = U A_s F\, \Delta T_{lm,CF} = 1805\ W$ Not a huge reduction!

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